A Formula for the Shaped Charge Jet Breakup-Time

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Eine Formel für die Breakup-Zeit eines Hohlladungsstachels

Die Breakup-Zeit eines Holladungsstachels, der aus einer rotationssymmetrischen Hohlladung entsteht, wird sehr genau wiedergegeben durch die Formel

$$t_B = \frac{1}{V_{PL}} \sqrt{8\,R \cdot T_L} \cdot \sin \beta/2$$

Diese Gleichung wird erhalten aus einer allgemeinen Grundformel, die hier zum ersten Mal angewandt wird und die besagt, daß die Breakup-Zeit bei homogenen, verformbaren Metallen unter sehr hohen Dehnungsgeschwindigkeiten gleich ist der kleinsten Anfangsdimension des sich verformenden Metalls dividiert durch die Geschwindigkeit V_{PL}. Bei Anwendung auf einfache Konfigurationen, wie z. B. auf ein Rohr, welches sich ausdehnt aufgrund einer Explosion im Rohrinnern, führt dieser Ansatz zu einer korrekten Voraussage der durchschnittlichen Größe der Bruchstücke. Mit diesem Ansatz wird zum ersten Mal eine Erklärung geliefert für das experimentelle Ergebnis, daß die Verformbarkeit von Metallen um eine Größenordnung zunehmen kann, wenn die Dehnungsgeschwindigkeit von 10^{-2} bis 10^5 pro Sekunde ansteigt.

Une formule pour déterminer le temps de fragmentation d'un jet de charge creuse

Le temps de fragmentation du jet engendré par une charge creuse à symétrie de révolution peut être déterminé de manière très précise au moyen de la formule suivante:

$$t_B = \frac{1}{V_{PL}} \sqrt{8\,R \cdot T_L} \cdot \sin \beta/2$$

Cette relation est déduite d'une loi générale, appliquée ici pour la première fois et qui précise que, pour des métaux homogènes et ductiles, soumis à des taux de déformation très élevés, le temps de fragmentation est égal à la plus petite dimension initiale du métal avant déformation divisé par la vitesse V_{PL}. L'application de cette loi à une configuration simple, par exemple à la dilatation d'un tube sous l'effet de la détonation d'une charge explosive à l'intérieur, permet de prévoir, de manière précise, la taille moyenne des fragments. Cette loi fournit pour la première fois une explication des résultats expérimentaux qui ont montré que la ductilité des métaux peut augmenter d'un facteur 10 lorsque le taux de déformation croît de 10^{-2} à 10^5 par seconde.

Summary

It was found that the breakup-time of jets formed by shaped charges of cylindrical symmetry is given very accurately by the formula:

$$t_B = \frac{1}{V_{PL}} \sqrt{8\,R \cdot T_L} \cdot \sin \beta/2$$

This formula is obtained from a general principal applied here for the first time which says that the breakup-time of homogeneous ductile metals under very high strain rates is equal to the smallest initial dimension of the elongating metal divided by V_{PL}. When applied to simple configurations such as a pipe which expands due to internal explosion this principal leads to a correct prediction of the average formed fragment dimensions. This principal provides for the first time an explanation to the experimental fact that metals' ductility can increase by an order of magnitude when the strain rate increases from 10^{-2} to 10^5 pro second.

1. Introduction

An exact prediction of the breakup-time of the shaped charge jet is essential for achieving an accurate and detailed description to the shaped charge performance. Careful measurements performed in B. R. L.[1] show that different parts of the jet break into segments at different times after the explosive ignition, the slower moving parts having longer breakup-times. An accurate calculation of e.g., the final jet radius must take this into account. We found that the most useful way for incorporating into computer calculations detailed breakup-time measurements such as those performed in Ref. 1 is to try to put them in the following formula:

$$t_B = (\sqrt{8\,R \cdot T_L} \cdot \sin \beta/2)/V_{PL} \tag{1}$$

where t_B is the breakup-time measured from the arrival of the explosion wave front at the liner element where the jet originates, R is the radius of the element, T_L is its thickness, β is the collapse angle and V_{PL} is the velocity specifying the liner material (the designation and the coordinates used are, as shown in Fig. 1, the same as in Ref. 2, i.e.: the X coordinate coincides with the liner axis of the symmetry, the origine $X = 0$ is at the liner apex and the positive direction points toward the liner base). V_{PL} is the velocity referred to in Ref. 1 as the average velocity associated with one segment into which the jet breaks. If e.g., a jet with a tip velocity of 8 km/s forms 50 segments in its part the velocity of which is higher than 3 km/s the average velocity associated with one segment is equal to:

$$V_{PL} = (8000 - 3000)/50 = 100[\mathrm{m/s}]$$

The statistical accuracy of the measurement of V_{PL} based on counting the jet segments is, in this typical example, about $\pm 100\,\sqrt{50}/50 \simeq \pm 14[\mathrm{m/s}]$.

When formula (1) is incorporated into a one dimensional computer code based on Ref. 2, such as the one described in Ref. 3, a very good agreement of the calculated breakup-time with the results of the measurements at Ref. 1 is achieved (see Fig. 2). We used for calculating the results at Fig. 1 the value $V_{PL} = 110$ m/s which is the average value of all the measurements of 10 shaped charges taken in Ref. 1.

Results of various measurements we performed since formula (1) was incorporated into a computer code were very accurately reproduced. V_{PL}, being the only free parameter in formula (1), can be measured either by segment counting or by fit of the calculations to the measurements. The last method gives the more accurate results.

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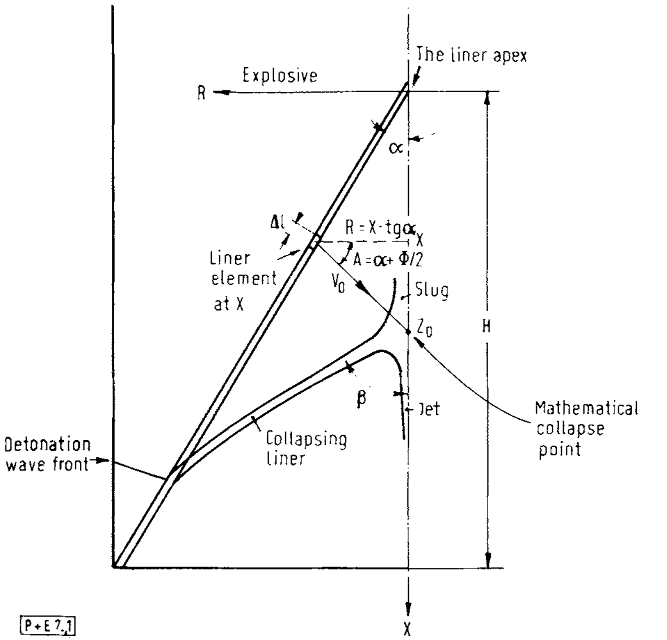

Figure 1. Illustration of the moment when jet and slug elements, originating at X, are formed.

It will be shown that finding formula (1) was not an accident. Although the physical explanation to its success is far from being established, formula (1) can be obtained by using relatively simple assumptions and considerations which may be successfully applied to various phenomena involving fast elongating homogeneous ductile metals.

2. Obtaining Formula (1) from Basic Assumptions

To obtain formula (1) two assumptions are made:
- The breakup-time t_B (defined above) is equal to

$$t_B = d_{in}/V_{PL} \tag{2}$$

  where d_{in} is the jet initial diameter (which is in fact its initial smallest dimension).
- The initial diameter is the diameter when the elongation starts.

Taking e.g. the case of a shaped charge with a conical liner we find (see in the Appendix) that the liner elongates during the collapse process in the direction of its formation line. The elongation of the material of which the jet is formed starts therefore at the moment when the liner is hit by the explosion wave front, because this is the moment when the collapse process begins. We therefore calculate d_{in} assuming the collapse process is a part of the jet elongation, which means that the jet material "remembers" its history before the jet was actually formed. We thus extrapolate the jet diameter back in time and calculate d_{in} as if the jet length upon formation were equal to the length of the formation line of the liner where it originates ("no elongation" condition).

Looking at the liner element of the length Δl in Fig. 1 we find its volume:

$$\Delta V_{EL} = 2\,\pi\,RT_L \cdot \Delta l.$$

Assuming no change in the liner material density happened, the part of this volume becoming a jet is, according to Ref. 2:

$$\Delta V_{EL} \cdot \sin^2\beta/2$$

calculating the jet diameter under the assumption of no elongation we write:

$$\pi\, d_{in}^2/4 \cdot \Delta l = 2\pi RT_L \cdot \Delta l \cdot \sin^2\beta/2$$

which leads by using formula (2) to formula (1).

3. Interpretation of Assumptions

To clarify the reasoning behind the two assumptions which lead to formula (1) let us examine another, more simple case where a metal elongates in a way similar to that of the shaped charge jet. A metallic pipe filled with high explosive is initiated at one end and photographed when a stable detonation is reached[4] (Fig. 3). The picture reveals the following three re-

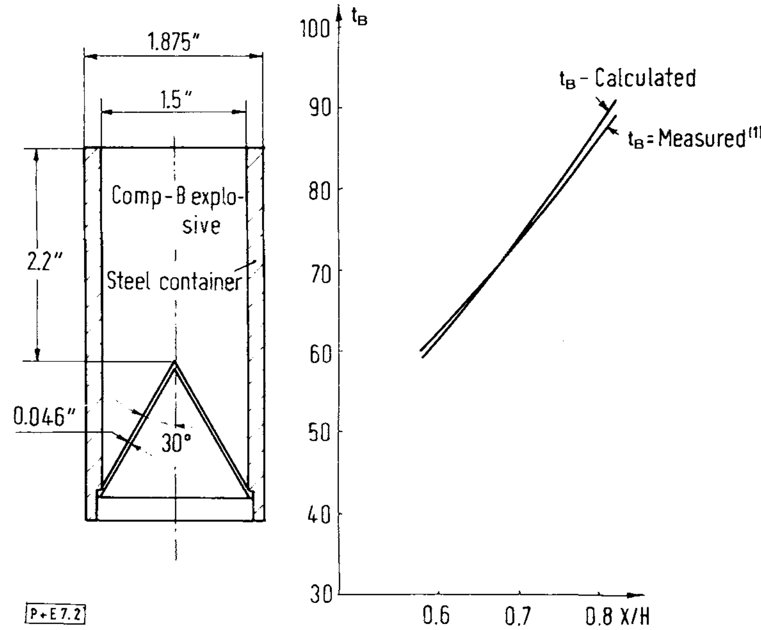

Figure 2. Comparison of the results, calculated by using formula (1) with the measurements of the breakup-time at Ref. 1. The jet belongs to the charge, with a copper liner, shown on the left. The breakup-time t_B is measured at Ref. 1 since the arrival of the explosion wave front at the liner cone apex. The time difference between this moment and the arrival of the explosion wave front at X was added to the breakup-time t_B obtained from formula (1) to obtain t_B as defined in Ref. 1.

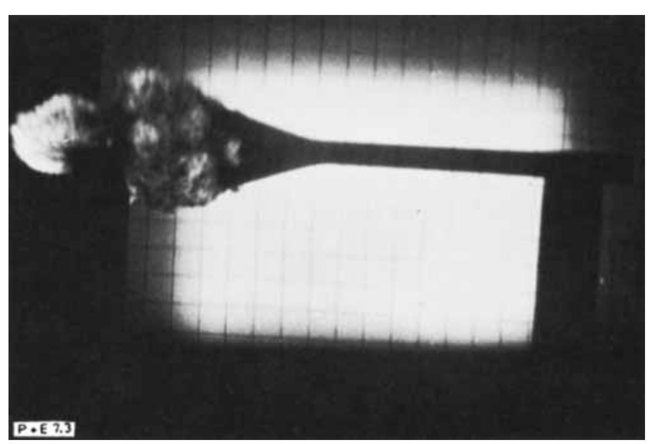

Figure 3. Pipe No. 4 at Table 1 expanding due to internal explosion of plastic PETN.

Table 1. Breakup-Time Measurements and Calculations of Aluminium Pipes' Walls

Exp. No.	Pipe Average Diameter [mm]	Pipe Wall Thickness [mm]	Calculated Breakup-Time (Using V_{PL} = 145 m/s)	Measured Breakup-Time Time [μs]
1	76.2	3.0	20.7	$18.0\ ^{+3}_{-1}$
2	76.2	3.0	20.7	$18.0\ ^{+3}_{-1}$
3	35.0	3.0	20.7	17.0 ± 2
4	20.8	1.2	8.3	$9.0\ ^{+2}_{-1}$
5	21.0	1.0	6.9	7.2 ± 1

gions: the part not yet detonated, the part where the pipe walls expand without being broken and the part where the pipe wall broke already into fragments and the gases produced by the explosion escape through the gaps formed between them. The length of the second part divided by the explosion wave velocity is equal to the breakup-time of the pipe wall measured since the arrival of the explosion wave front at the pipe element which brakes exactly at the moment the pipe is photographed. We decided to perform this simple experiment with few aluminium pipes (Table 1) and soon found that the breakup-time of the pipe wall could be predicted by the formula:

$$t_{BP} = T_p/V_{PL} \tag{3}$$

where T_p is the pipe wall thickness and V_{PL} = 145 m/s is the average value of those measured by B. R. L.[1] using aluminium shaped charge jet segment counting.

Since no check was made whether the aluminium used in all the experiments (including those in Ref. 1) was metallurgically identical, the accuracy of the prediction was not expected to be high. Furthermore it was difficult to measure the avarage value of t_B and therefore the minimum value was taken (see Fig. 3). Still the results are convincing.

We allow ourselves to compare formulas (2) and (3) by saying that the shaped charge jet elongation can be looked upon as an elongation of a sector of a ring (element of the pipe) with a radius which is very big comparing to the length of that sector. The initial thickness of the jet, d_{in} and the ring T_P correspond to one another because it is the smallest initial dimension which determines the breakup-time, other details of the cross section (rectangular or circular) being of negligible influence comparing to the accuracy of the measurements performed so far.

The reason why formulas (2) and (3) seem to describe the same phenomenon in spite of many appearent differences of the pipe and jet configurations, stems from the fact that the metal is brought to its final velocity within an acceleration time which is very small comparing to its breakup-time[4]. During most of the time the metal elongates with no significant forces acting on it other then its own yield strength, and the inertial forces stemming from the state of streaming it was initially brought into by the explosion. Note that in the cases we examined the metal yield stress σ_y is too small to effect its velocity to a measurable amount.

It should be noticed that formulas (2) and (3) hold both, using the same V_{PL}, in spite of the fact that the pipe wall becomes thinner then the jet diameter when they both undergo the same elongation, due to the difference between (assuming constant density) linear and aerial shrinkage. This points to the fact that the initial thickness of the metal is the only magnitude suitable to be used for calculating the breakup-time, (which leaves no alternative to the second assumption). When the elongation process is already in progress its history is stored in the metal in a way which we cannot measure just by looking simply at the metal thickness. It can be guessed that the shape of streaming inside the metal influences the way in which the phenomenon responsible for the final breakup developes.

4. Physical Interpretation

We go a step further and illustrate a schematic picture to clarify the possible nature of the physical process which determines the breakup-time.

Suppose disturbances, responsible for the final breakup, start to develope at the liner surfaces (both inside and outside) immediately upon its being hit by the explosion wave front. Suppose these disturbances propagate with the velocity V_{PL} into the metal until some of them reach the other side of the elongating metal thus breaking it along their propagation line, while the rest cease to propagate because the broken metal stops to elongate[5]. The directions of propagation of the disturbances are not far from being random even though we expect the shape of streaming inside the metal to have a general influence on their trend. Those which happen however to reach the other side first, determine the breakup-time.

Such a picture implies that t_B is not just a measurable number but is really a statistical distribution, i.e.: if we measured the breakup-time of a specific jet element (e.g. that the velocity of which is between 5 km/s and 5.5 km/s) of many identical shaped charges we would not get the same result. We would find instead a distribution of results the average of which is given by formula (1). The detailed measurements at Ref. 1, in fact, confirm this consequence. They reveal a distribution with a large width, (the standard deviation of which is about $t_B/4$) which is expected from a mechanism having a big random factor such as the one illustrated above.

Knowing the specific physical nature of the propagating disturbances is not crucial for achieving a sufficiently accurate mathematical description of the breakup-time phenomenon. It is convenient however to regard them as a development of fractures the typical shape of which is found by looking at the edges of natural fragments (Fig. 4). The edges of segments which shaped charge jets form are, as found by taking a careful look into X-Ray photographs, of the same nature, (Fig. 5).

Note that since the jet is formed from the inside of the liner (the side opposite to the explosive) the disturbances which break it (if the suggested description is true) start at this surface while those starting on the outside go to the slug.

5. Linear Charges

When the development leading to formula (1) is repeated for linear charges the following formula for the linear charge's jet breakup-time is obtained:

$$t_{BLC} = \frac{2T_L}{V_{PL}} \sin^2 \beta/2 = \frac{T_L}{V_{PL}} (1 - \cos\beta) = t_{BL} (1 - \cos\beta) \tag{4}$$

Here t_{BLC} means breakup-time of linear charge and t_{BL} is the breakup-time of the liner assuming no jet were formed. If we rewrite formula (1) using t_{BL}, formula (5), a big difference

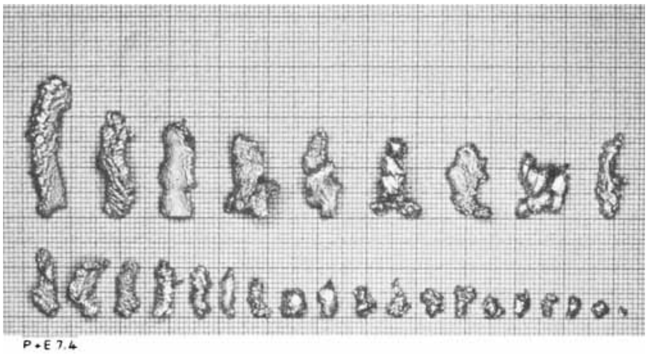

Fig. 4. Typical fragments of pipe No. 3 (in Table 1). The calculated average fragment width is 4.2 mm. The fragments are put on a 1 mm × 1 mm square paper.

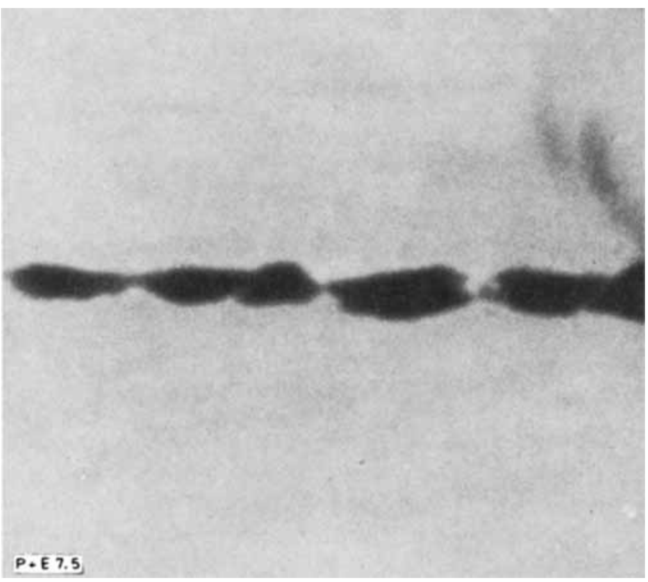

Figure 5. Illustration of an X-ray photograph showing the breakup moment of a part of a jet.

between the linear and cylindrical shaped charge configurations shows up:

$$t_{BCC} = 2 \cdot t_{BL} \quad \sqrt{\frac{R}{T_L}(1 - \cos\beta)} \tag{5}$$

While the jet breakup-time is in practically used shaped charges of the cylindrical configuration few times bigger then t_{BL} it is always smaller then t_{BL} in the linear charges. A coherent jet can be formed only if[3, 6] the collapse duration T* is smaller then t_{BL} (otherwise the liner breaks before a jet is formed). T* can however be close to t_{BL} i.e.: it can happen that a jet is formed by a linear charge but $t_{BLC} < T^* < t_{BL}$.

This appearent contradiction is the outcome of our neglecting the fact that when t_{BL} is small enough comparing to T* the real breakup-time gains from an increase in T* i.e., from a delay in forming the jet. To take this effect into account the following method was introduced for practical calculations:

If $t_{BLC} \geqslant 2 \cdot T^*$ use formula (4). In the complementary case, $t_{BLC} < 2 \cdot T^*$ redefine the breakup-time as follows:

$$t_{BLC\ \text{redefined}} = T^* + ½\, t_{BLC} \tag{6}$$

by this formal method a continuity between the regions of T* smaller and bigger then ½ t_{BLC} is achieved.

The experience in using formula (6) for linear charges has so far given reasonably accurate results, may be because T*, which does not depend on the breakup-time model was big comparing to ½ t_{BLC} in the cases to which formula (6) was applied. It can therefore be concluded that breakup-time calculations using formulas (4) and (6), as well as formula (1) with a similar redefinition for very large collapse durations, is expected to be reliable for breakup-times which are either few times bigger or few times smaller than 2 T*, while the intermediate region has still to be studied.

6. Natural Fragmentation of Homogeneous Ductile Metals

The average number of segments, the shaped charge jet breaks into, is given by the formula:

$$n_j = \Delta V_j / V_{PL} \tag{7}$$

where ΔV_j is the velocity difference between the two edges of the jet. The number of fragments formed on a circumference of an expanding pipe is similarly equal to

$$n_p = 2\pi V_o / V_{PL} \tag{8}$$

where V_o is the average radial expansion velocity[4]. The circumference of the pipe is equal, when it breaks, to

$$L_B = 2\pi(R_o + V_o \cdot t_B)$$

the average fragment width W_F is therefore obtained by dividing L_B into n_p. Using formula (3) we thus get:

$$W_F = T_p + R_o \cdot V_{PL}/V_o \tag{9}$$

The thickness T_F of the same fragment will be:

$$T_F = T_p \cdot \frac{R_o}{R_o + V_o \cdot t_B} = T_p \Big/ \left(1 + \frac{V_o \cdot T_p}{V_{PL} \cdot R_o}\right) \tag{10}$$

The length of the fragment tends, according to formula (9) to be large since the radius of curvature, which stands for R_o, along the formation line of the pipe tends to infinity. In practice however, fractures of the pipe walls are not parallel but tend to cross one another thus limiting the fragment length to few times their width. Typical fragments of pipe No. 3 in Table 1, caught in a water pool are shown in Fig. 4.

Our measurements of fragments' average dimension so far, fully confirmed formulas (9) and (10). The more general form of formula (10), used for ellipsoidic shells, having two radii of curvatures R_1 and R_2 is:

$$T_F = T_p \Big/ \left[\left(1 + \frac{V_o \cdot T_p}{V_{PL} \cdot R_1}\right) \cdot \left(1 + \frac{V_o \cdot T_p}{V_{PL} \cdot R_2}\right)\right]$$

In formula (9) R_1 and R_2 are submitted for R_o to get the respective average fragment dimensions.

7. Discussion

It was found and confirmed by experiment that the breakup-time of shaped charge jets as well as pipes' shells expanding

under internal explosions is given, at least for some homogeneous ductile metals by the general expression:

$$t_B = T_{min}/V_{PL} \quad (11)$$

Where T_{min} is the smallest initial thickness of the elongating metal and V_{PL} is a velocity which specifies it. It may be attempted to identify this velocity with the expression:

$$V_{PL} = \sqrt{\sigma_y/\varrho} \quad (12)$$

known as the plastic velocity where σ_y is the metal dynamic yield stress and ϱ is its specific density. The identification is hard to proove experimentally because of the difficulty in measuring σ_y accurately enough under the high strain rates characterizing the jet or pipe wall elongation (the strain rate of the pipe wall is equal to V_o/R_o; the strain rate of a jet formed by a typical shaped charge the radius of which is also R_o, is almost the same obtained for the pipe wall).

It is interesting to note that the fact that shaped charges obey the scaling laws (see e.g. Ref. 7) implies that V_{PL} does not depend on the strain rate of the metal at the region where the laws were confirmed. If formula (12) is true, then σ_y must also be practically a constant throughout this strain rate region.

By finding whether σ_y changes considerably or not wherever the scaling laws hold, it is therefore possible to support or to disprove formula (12). One should notice on the other hand that σ_y is expected to increase upon reduction of the jet temperature. It is experimentally found that shaped charges' performance reduces considerably when used at temperatures as low as $-50\,°C$. Under such initial conditions the final jet temperature[8] is also expected to be lower then usual. Due to formulas (12) and (1) this causes a reduction in the jet breakup-time. Formula (12) can therefore provide an explanation to this experimental fact. (Note as well that the results of the "Hemp" computer code calculations presented in Ref. 1, even though based on a different model then presented here are also consistant with the trend implied by formula (12) i.e.: either an increase in σ_y or decrease in ϱ will shorten the breakup-time). The question whether the identification (12) is true or exact is interesting because V_{PL} seems to be a key quantity to the behaviour of the metal in all the phenomena which involve high strain rate elongation. In the examples discussed above it seems even to be the only one. The use of rule (11), on the other hand, does not depend on the specific answer to this question. Thus, whenever the problem we deal with is of a geometrical nature and not of a metallurgical one V_{PL} is needed only as a constant which describes the specific metal used.

The concept of fractures or disturbances which propagate through the metal with the velocity V_{PL} (or a velocity proportional to but close to it) is consistant with the observation that the breakup-time is not just a measurable number but a wide distribution of which t_B is the average value. This concept has not yet been used however, to develope a detailed mathematical model and all the formulas obtained here do not depend on such a model. A complete mathematical model based on such a concept will have to reproduce the distribution of the lengths of the segments the shaped charge jet breaks into, the distribution of the pipe wall fragments' dimensions, the breakup-time distribution etc. The result is expected to be close to Mott's[5] distribution normalized to the fragments' average dimensions calculated by the method described above. This method deals in fact to a great extent, with questions which remain open in Mott's basic work. The idea that the time available for the elongation and not the elongation itself is the parameter which dominates the metal ductility at very high strain rates explains e.g. for the first time in a simple way the experimental observation that the ductility of certain metals (e.g. copper and aluminium) can increase by an order of magnitude (from nearly 30 to nearly 200 percents) when the strain rate increases from 10^{-2} to 10^{+5} pro second.

To illustrate the dominancy of the time as a basic parameter of the elongation process let us refer to the following specific example: By using a one dimensional computer program similar to the one described in Ref. 3 into which formula (1) is incorporated it was found that the elongation η_B of the shaped charge jet at its breakup (see in the Appendix for details) is proportional to the jet velocity gradient V_j' times the breakup-time t_B. For the standard B. R. L. 81.3 mm, 42° opening angle shaped charge taken as an example the following relation was found to hold:

$$\eta_B = t_B \cdot (dV_j/dX) \cdot (0.84 \pm 0.01) \quad (13)$$

This relation holds within the above accuracy (which also depends on the accuracy of the mathematical model[2] used in the computer code) while η_B changes from about 2 near the jet tip up to above 16 at the rear (slower) part of the jet. Since the temperature along the jet can be assumed to be close uniform[8] (it is determined mainly by the explosion wave front pressure which is constant) its effects on the metal ductility do not interfere and a relation as simple as Eq. (13) could be reached.

8. Summary and Conclusions

The breakup-time of a shaped charge jet element measured from the arrival of the explosion wave front at the liner element where it originated is given by formula (1) with a very good accuracy. The liner metal is characterized by one parameter, V_{PL} which could perhaps be identified with the plastic velocity, formula (12). Formula (1) is obtained from a general principal which says that the breakup-time at high strain rates is equal to the smallest initial dimension (thickness) of the elongating metal divided by V_{PL}, formula (11). This principal, introduced here for the first time, provides a simple explanation to the experimental observation that the ductility of metals like aluminium and copper can increase by an order of magnitude when the strain rate applied to them is increased from 10^{-2} to 10^{+5} pro second. This explanation is supported by our finding that the jet elongation at breakup is proportional to the jet velocity gradient times the breakup-time.

When this principal is applied to simple configurations such as an expanding pipe made of a homogeneous ductile metal, it leads to a correct prediction of the average fragment dimensions. In more complicated cases e.g. that of a linear charge valuable information can also be obtained if the principal is applied with care.

The assumption that natural fractures propagate almost randomly from the liner surface with the velocity V_{PL} until they reach another free surface is consistant with the breakup-time detailed measurements at Ref. 1 which show that the shaped charge jet breakup-time is widely distributed around the average value predicted by formula (1). The jet overall length is not highly effected by the width of this distribution because it consists of the sum of the segments' lengths and the differences

in the times in which the segments were formed (when the breakup at the segment's both sides ends) tend statistically to compensate for one another.

9. References

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Appendix

Elongation of the liner along its formation line during the collapse process

X-ray photographs of collapsing shaped charge liners clearly reveal that the outer periphery of the liner (at its base) detaches at the end of the collapse process from the rest of the liner and forms a ring which is neither a part of the jet nor of the slug (see e.g. Ref. 9, p. 203). The formation of such a ring is the result of a considerable elongation of the liner near the base in the direction of its formation line. A detailed calculation based on the method for describing the collapse process of Pugh, Eichelberger and Rostoker[2] shows in fact that the liner elongation during the collapse process along its formation line is small near the cone apex but becomes ever bigger toward the base (see Fig. 6).

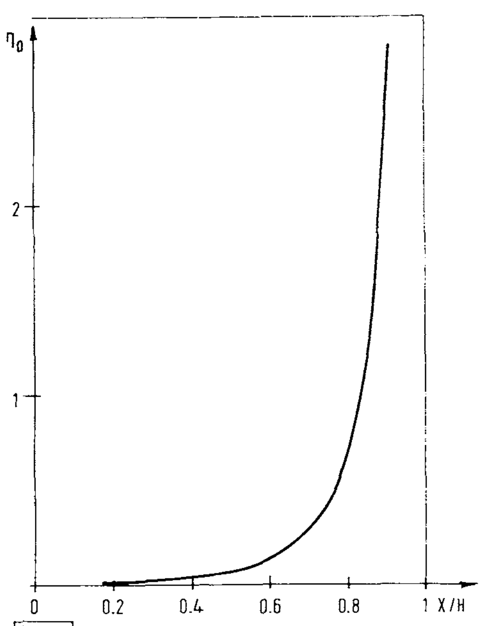

Figure 6. Elongation of the liner of the standard 42° opening angle 81.3 mm, B. R. L. shaped charge, during the collapse process, in the direction of its formation line, calculated by formula (A-5).

The basic points of the calculation are as follows:
The location of the collapse point Z_o on the axis of symmetry of the shaped charge (see Fig. 1) is given by the expression:

$$Z_o = X + R \cdot \mathrm{tg}A \qquad \text{(A-1)}$$

where $A = \alpha + \Phi/2$ (Φ is the angle by which the liner tilts upon passage of the explosion wave at the liner element[2]).

At the time t, later then the collapse moment, the jet which originated at X reaches the point[3]:

$$\xi(X, t) = Z_o + V_j(t - T - T^*) \qquad \text{(A-2)}$$

where V_j is the formed jet velocity, T is the time ellapsing between the initiation of the explosive (defined t = 0) and the explosion wave front arrival at X, and T^* is the liner element collapse duration[2,6], (quoting Ref. 2: $T^* = R/(V_o \cdot \cos A)$; V_o is the liner velocity).

A liner element between X and $X + \Delta X$ has a formation line element the length of which is equal to: $\Delta l = \Delta X/\cos\alpha$.

The length of this element at the time t becomes: $\Delta\xi = \xi(X, t) - \xi(X + \Delta X, t)$. The elongation $\eta(X, t)$ of the liner material at the time t is therefore equal to

$$\eta(X, t) = \lim_{\Delta X \to 0} \frac{\Delta\xi}{\Delta l} - 1 = -\cos\alpha \frac{\partial\xi(X, t)}{\partial X} - 1 \qquad \text{(A-3)}$$

Taking the derivative of formula A-2 and substituting in A-3 we finally get:

$$\eta(X, t) = \eta_o(X) - V_j'(t - T - T^*)\cos\alpha \qquad \text{(A-4)}$$

where

$$\eta_o(X) = -1 - \{1 + R' \cdot \mathrm{tg}A + RA'/\cos^2 A - V_j[T' + R'/V_o \times \cos A) - RV_o'/(V_o^2 \cdot \cos A) + RA' \cdot \sin A/(V_o \cdot \cos^2 A)]\}\cos\alpha \qquad \text{(A-5)}$$

η_o is equal to the sum of all the terms which do not depend on t. At the collapse moment $t = T + T^*$ and according to Eq. A-4: $\eta(X, t) = \eta_o(X, t)$.

Fig. 6 shows $\eta_o(X)$ computed for the standard above mentioned B. R. L. shaped charge. The values this function gets near the liner base easily clarify how the above mentioned ring is formed.

Note that $\cos\alpha = \cos 21° = 0.9336$ for the standard B. R. L. shaped charge is not equal to the constant 0.84 ± 0.01 appearing in Eq. (13). Equation (13) therefore is not implied by the mathematical model but is at least to some extent a result of the substitution of t_B expressed in formula (1) into Eq. A-4 to calculate η_B.

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